SIMULATION OF INSPECTIONS OF ELASTIC WAVEGUIDES OF ARBITRARY SECTION CONTAINING ARBITRARY LOCAL DISCONTINUITIES OR DEFECTS

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ABSTRACT. Optimizing guided wave (GW) examinations or interpreting GW measurements can be greatly helped by simulation tools dealing with complex propagation and scattering phenomena. Tools are developed exploiting the modal nature of GW, this simplifying interpretation (no postprocessing as required when results are computed without reference to modes). Two modal formulations simulate measurements (pitch-catch, pulse-echo); they link a semi-analytic finite element code for computing modes, models for transducer diffraction and a specific finite element model for scattering by arbitrary discontinuities. The paper reviews these tools; examples illustrate their interest.

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INTRODUCTION

Elastic guided waves (GW) propagate at long range in the thickness of parts of regular shape which thickness is of the same order of magnitude as wavelengths. This property is very attractive for the nondestructive evaluation (NDE) of large structures since it limits or even avoids transducer scanning; this reduces the overall duration and cost of the examination and makes its implementation easier [1,2]; GW are also measured in NDE by Acoustic Emission (AE) of pressure vessels and can be passively and actively used in Structural Health Monitoring (SHM). Other intrinsic properties of the physical behavior of GW tend to lessen their interest. i) most GW are dispersive – their speed is frequency dependent, ii) they are multi-modal – at a given frequency, several modes coexist, their number growing with frequency; iii) modes couple when interacting with a discontinuity of the guide; iv) since their wavelength compares with structure thickness, spatial resolution is limited. All these characteristics make difficult the interpretation of results as well as the design of optimal testing configurations. Simulation tools can constitute the appropriate mean to overcome these difficulties and are expected by industrial conceivers of GW inspections. It is our objective to address these industrial needs.

In this paper, we first review our modeling approach adopted for simulating NDE methods involving GW propagation and the tools used or developed at CEA to implement

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it; mathematical demonstrations are not given but due references are proposed to interested readers. Then, the advantages of the simulation approach are illustrated by examples of NDE examinations involving GW propagation. Works in progress or future are mentioned.

THEORY

Modeling Approach

Since most guided waves are dispersive, GW testing is operated in general in a limited frequency bandwidth; excitation signals are typically in the form of a single frequency (CW) signal modulated in amplitude (Gaussian wave packets, tone bursts etc.). It is therefore natural to model GW in the frequency domain, typical waveforms measured being synthesized by Fourier transform over a limited spectrum. In what follows, models are described under CW assumption; ω denotes the angular frequency.

A first key property of GW propagation is that at a given frequency, GW can be decomposed as (complex-valued) linear combination of eigenmodes peculiar to the section of the structure (perpendicular to its guiding axis denoted by z) and to its stiffness. The knowledge of the set of modes and their behavior is sufficient to depict the wave behavior of any elastodynamic quantity (particle displacement or velocity, stress) relative to an arbitrary field. The nth mode of this set is described at a given frequency by: i) its wavenumber β_n, real for the finite number of propagative modes, imaginary for the finite number of evanescent modes or includes an imaginary part for the infinite number of inhomogeneous modes, ii) the corresponding particle displacement vector in the invariant section of the guide $\tilde{\mathbf{u}}_{\mathbf{n}}(x, y)$. The CW displacement $\mathbf{u}$ associated to a wavefield writes

$$\mathbf{u}(x, y, z; t(\omega)) = \sum_n A_n \tilde{\mathbf{u}}_{\mathbf{n}}(x, y) e^{j(\beta_n z - \omega t)}, \quad (1)$$

where A_n denotes the nth amplitude coefficient in the decomposition. In practice, the knowledge of mode behavior and dispersion characteristics is an essential step for understanding complex phenomena arising in a guiding structure. Measured or simulated signals are then interpreted in reference to modes: typical questions concern the ability of the modes to be transmitted through or reflected on a guide discontinuity, to be converted into other modes in the interaction etc. If simulated results are computed regardless of the modal nature of GW, they are very often post-processed at a considerable computation cost to be eventually interpreted as variations of mode amplitudes. Therefore, we made the choice to develop simulation tools fundamentally on the basis of the modal description of waves in each portion of the structure that propagates GW.

A second key property of GW propagation – as applied to NDT, is that it can be described as a global phenomenon in the homogeneous portions of the structure but the way GW interact is otherwise dominated by local phenomena (e.g., transducer diffraction both in radiation and in reception, scattering by a defect, by a variation of geometrical or material properties of the structure and by any inhomogeneity of the structure). GW are especially attractive for their ability to propagate over large distances; NDT is operated by transducer(s) and aims primarily at detecting defects which are localized. Thus, the GW/NDT simulation tools must deal with various scales corresponding to various phenomena. Moreover, industrial needs for simulation suppose that tools can be used intensively. A single method cannot be effective for both local and global computations. We believe that various phenomena at different scales require various models. A further ingredient is necessary to give these models the possibility to work all together at the same time and in synergy. Next paragraph recalls the overall formulations [3] that constitute this

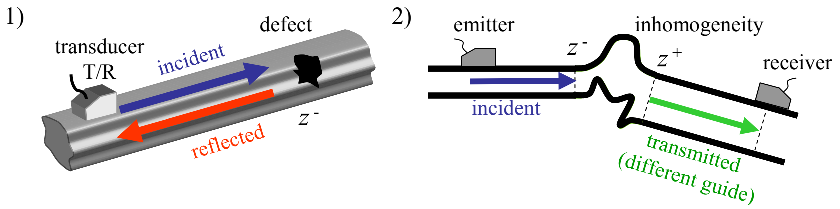

FIGURE 1. 1) Configuration where a single transducer is used in the transmit-receive mode. 2) Configuration where two separated transducers (one emitter, one receiver) are positioned on two different waveguides.

central ingredient, followed by the description of models used or developed for computing modes, transducer diffraction effect on them and their scattering by inhomogeneities.

Overall Modal Formulations [3]

Two configurations (denoted by 1) and 2), see Fig. 1) are considered for which two formulations were derived. The first (resp. second) is a pulse-echo (resp. pitch-catch) configuration. The two frequency-dependent expressions (Eqs. 2) of the signal received $s_i(\omega)$, i=1, 2, were obtained using some mathematical properties of guided modes (bi-orthogonality) and the electro-mechanical theorem of reciprocity proposed by Auld [4].

$$s_1(\omega) = \frac{-i\omega}{P} \sum_{n\in\mathbb{N}} \sum_{m\in\mathbb{N}} A_n^r A_m^e R_{nm} e^{i(\beta_n+\beta_m)z^-}, \tag{2a}$$

$$s_2(\omega) = \frac{-i\omega}{P} \sum_{n\in\mathbb{N}} \sum_{m\in\mathbb{N}} A_n^r A_m^e T_{nm} e^{i\beta_m z^-} e^{i\beta_n (L-z^+)}, \tag{2b}$$

where P is the electrical power provided to the emitter. A_m^e and A_n^r are respectively the amplitude of mode m radiated by the transducer and the amplitude of sensitivity to mode n of the transducer in reception; they stand for transducer diffraction effects. R_{nm} (resp. T_{nm}) is the reflection (resp. transmission) coefficient for the incident mth mode and the reflected (resp. transmitted) nth mode; they stand for the scattering by an inhomogeneity of the guide(s). z^- denotes the distance between the emitter and the scattering zone; $L - z^+$ denotes the distance between the scattering zone and the receiver in configuration 2; these distances appear together with β_n, the wave number of the nth mode of a given guide, in exponential terms which are propagators of GW in the various guides involved. In practice, as soon as transducers are sufficiently distant from the scattering zone, the two discrete sums in equation (2) can be restricted to the sole propagative modes.

In these formulas, the various local phenomena (causing variation of mode amplitude) and the global propagation in homogeneous guides are mathematically separated. These formulas can admit different methods for computing the various terms of the double discrete sums. Another crucial point about them is that they make it possible to combine existing results for some of the terms with new results for other terms, opening onto vast post-processing capabilities. It is possible to use a scattering matrix with several amplitudes relative to different transducers without re-computing the whole simulation; this constitutes an economical way of optimizing testing configurations. Examples of such capabilities will be given.

Mode Computation by the Semi-Analytic Finite Element Method

There are many methods in the literature for computing modal solutions; some are more appropriate than others for a given application. Our aim being to offer generic tools, the semi-analytical finite element method (SAFE method, see [5] for example) appeared to be very well suited to our needs. It is very well documented (notably in this series) and is still under developments for adding new capabilities. In few words, this method involves a finite element computation in the guide section, allowing the computation of both wave vectors and modal displacements in the section as being the eigenvalues and eigenvectors (resp.) of a quadratic system of equations; this system is the discrete form of a variational problem in the guide section. Since this is a finite element computation, it allows one to deal with all sorts of characteristics of the guides (section shape, constitutive materials). As it is restricted to the section, it is computationally very efficient. The propagation is otherwise accounted for by means of analytic propagators in the guiding direction normal to the section considered, at no computational cost (same function whatever the range). In Eqs. (2), this model gives the solution for the exponential propagator terms.

Local Models of Transducer Diffraction

There are basically two cases to distinguish: the transducer(s) is positioned, i) on the guiding surface; ii) on the guide section. Each case requires a specific model for computing the amplitude of the modes. Then, it is necessary to derive models adapted to the transduction that takes place, this depending on the type of transducer used (piezoelectric, EMAT, magnetostrictive). Again, this problem is addressed in the literature for various cases of industrial interest. In the present implementation, our models deal with piezo-transducers assumed to be sources of normal stresses all over their active surface. For transducers acting from the section, a specific variational formulation has been derived which is discretized on the elements used for computing modes by SAFE [6]. The case of nonuniformly excited transducers – the applied stress is made variable along the active surface – has been treated allowing us to propose two methods for selecting one single mode chosen among possibly many modes [7]. For those acting from the guiding surface, a surface integration over the transducer area must be computed. In the case of an angled probe, which is a very common way of selecting a mode at a given working frequency by phase coincidence, it can even be computed analytically [8]. In all cases, the results are expected to be given in the form of a linear combination of modes, as in both references cited here [6, 8]. In Eqs. (2), these models give the solution for the terms A_m^e and A_n^r.

Local Models of Scattering by Defects and by Guide Discontinuities

Computing the scattering by a guide inhomogeneity is a difficult task. Contrary to bulk waves typically used in NDT, guided waves have in essence a wavelength comparable with the dimensions of the guide section and the size of the inhomogeneity. In the former case, scattering can be accurately computed by means of approximations (high frequency); in the latter, deriving suitable approximations is almost impossible.

Our aim is to write the solution of the scattering problem in the form of a matrix of complex coefficients – the reflection R_{nm} and transmission T_{nm} coefficients in Eqs. (2), assuming that modal solutions in all the guiding structures connected to the local zone of scattering are known. This matrix links an input vector constituted by the coefficients of decomposition of the incoming wave in its guiding structure, to output vectors constituted by the coefficients of decomposition of the outgoing waves in their guiding structure.

An efficient method was proposed [3] for planar crack-like defects of arbitrary shape in an otherwise homogeneous guide, assuming that the crack surface belongs to the guide cross-section. By taking advantage of the symmetry, a variational formulation was

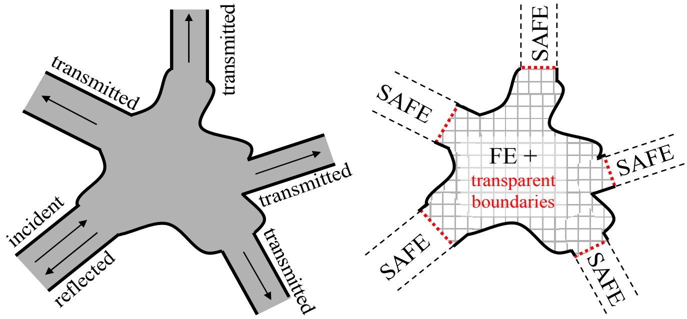

FIGURE 2. Left: scattering by the junction of several guides. Right: the solution involves SAFE computations in the connected guides and a FE computation including transparent boundaries in the junction.

derived discretized on the elements of the SAFE calculation.

To deal with arbitrary flaw shapes or guide inhomogeneities (section or material variations, junction as shown in Fig. 2, etc.), an original finite element (FE) scheme has been developed with the further goal to limit the computation zone to a minimal size for efficiency (full demonstration in [9], extended review in [10]). The computation zone being necessarily of finite size, its boundaries with all the guiding structures connected to it must be transparent for elastic waves: they must not reflect the incoming waves nor reflect outgoing waves. Thus, the main task in this development was the obtaining of artificial boundary conditions endowing transparency. Radiation condition at infinity is brought back to the artificial boundaries by building an operator coupling the finite elements inside the zone to the modal solutions in guides. The operator combines the displacement components with axial stresses (axes of the various guides) – Dirichlet-to-Neuman type. By doing so, an original mixed variational formulation was derived combining the displacement and a multiplyer associated to the axial stresses. The scattered field is projected on modal solutions in guides through the use of bi-orthogonality relations expressed for all guides, this being done while solving the FE system. As the new system partly relies on classical FE discretization of elastodynamics, it is straightforward to introduce internal sources inside the FE zone; an example of application for simulating an examination by the Acoustic Emission method will be given for illustrating this possibility.

EXAMPLES OF APPLICATION TO NDT

Two applications to NDT are treated here to illustrate possible uses of the tools described. In the first, the scattering by a junction of three identical guides is considered; post-processing capabilities are used to study the effects of mode selection on measured waveforms. In the second, internal forces are introduced that model a source of AE at the tip of a crack; the field radiated is decomposed into modes propagating in the structure.

Reflection from and Transmission through a Complex Junction

The junction considered (Fig. 3) is that of three plates (40-mm-thick) in steel. The radiation is operated from plate #1 and reflection and transmission from and through the junction in plates (#1-3) are studied. The scattering matrix is computed over the bandwidth [49.5–63.5] kHz. Figure 4 shows the total field inside the junction for an incident S0 mode at three frequencies (lowest, center, highest). The FE computation results in three scattering matrices (reflection, transmission) for the various possible incident modes (three propagative modes in the frequency range) scattered as propagative modes in the guides.

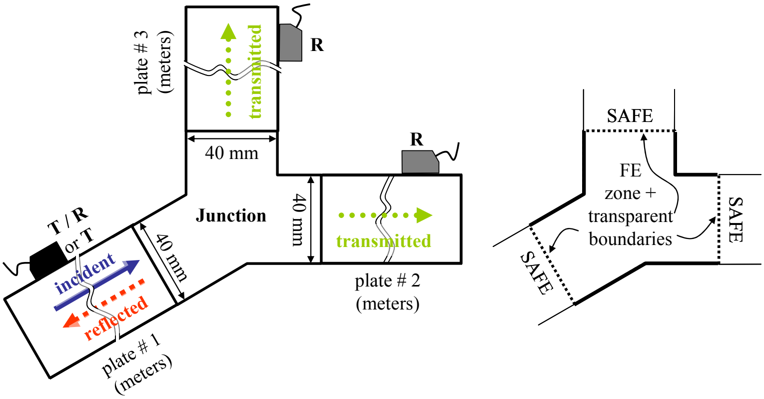

FIGURE 3. Left: The junction considered in the computation. Angled-probes are used as T and R on the various guides, allowing several configurations to be processed. Right: elements of the computation.

Each matrix is a 3x3 (for 3 propagative modes) of frequency dependent coefficients. Figure 5 shows such a result for S0 incident mode. Equivalent results were obtained for other incident modes but the lack of space prevents us to show them here. It may be noticed that the sum of squared coefficients equals 1 (conservation of energy). Results for the scattering matrices are now processed by means of the overall formulas [Eqs. (2)], for various transducers. We consider angled- probes, one in T/R mode on plate #1 and two receivers on plates #2-3, 2 meters away from the junction. Shoe angles are taken so that phase coincidence occurs between a bulk CW in the shoe and a chosen mode at the center frequency (56.5 kHz): 70.6°, 25.2°, 19.5° for A0, S0 and A1. The active length equals 50

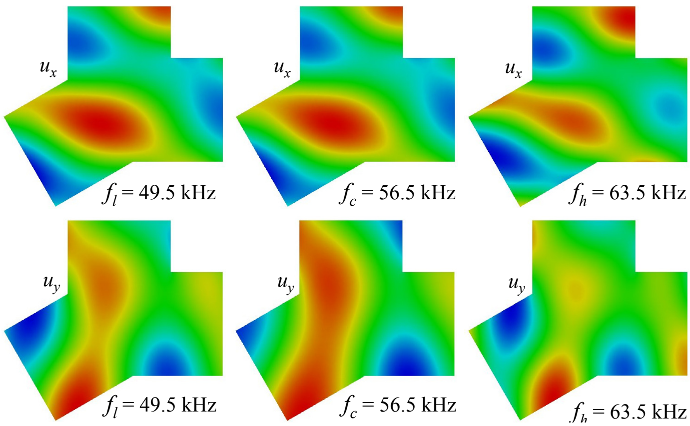

FIGURE 4. Total displacement inside the junction at different frequencies for S0 incident mode.

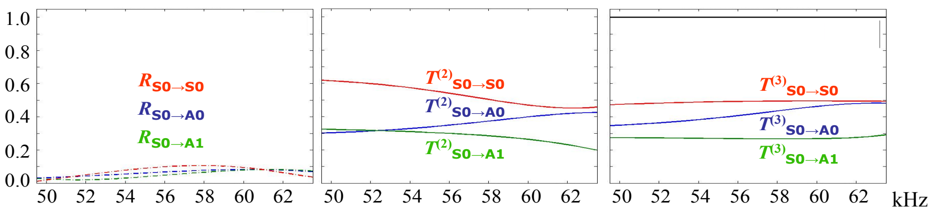

FIGURE 5. For S0 incident in plate #1, scattering coefficients in plates #1-3 as functions of frequency (kHz). Left: reflection in plate #1 – Center: transmission in plate # 2 – Right: transmission in plate #3.

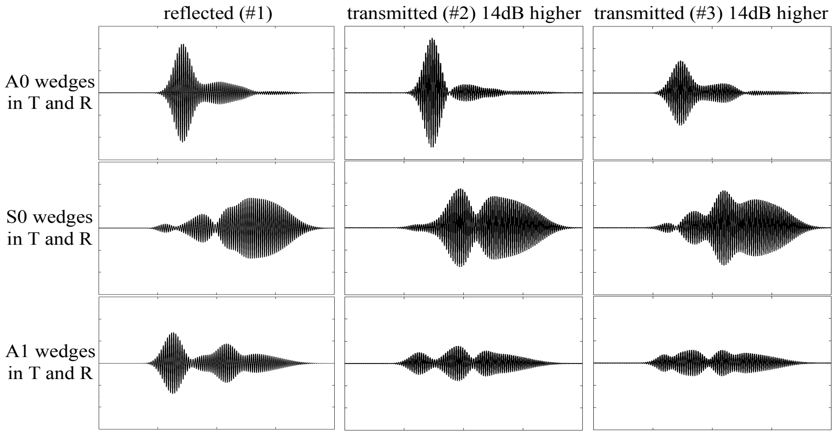

FIGURE 6. Waveforms received in plates #1 (L), #2 (M), #3 (R), for the 3 wedges. Time-scale [1.5-3.0] ms.

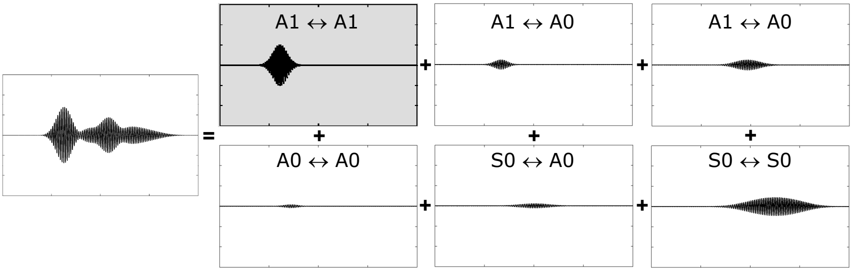

FIGURE 7. Reflected signal with A1 probe made of 9 elementary signals (3 equal to 3 others by symmetry).

mm. The excitation pulse is Gaussian (10% bandwidth at -6dB). Waveforms simulated are shown on Fig. 6. Transmitted ones (middle, right columns) of higher amplitude than reflected ones (left) are displayed with a 14 dB scaling factor. In Eqs. (2), each term of the double sum can be seen as a signal itself. Therefore, one can easily interpret signals by decomposing them into elementary signals as shown by Fig. 7.

Acoustic Emission by a Crack

The hybrid model is used for decomposing the field radiated by a AE source on modes over the narrow bandwidth of a AE probe. AE sources are generally transient. However, due to the low amplitudes radiated, transducers used as receiver of possible AE events are commonly resonant transducers (high sensitivity but very limited bandwidth). The configuration considered (Fig. 8, top) is that of a 20-mm-thick steel plate subjected to external load, including a 45°-tilted surface-breaking crack. The crack tip is assumed to be source of a transient force aligned with the crack surface. The modal decomposition of the field radiated by the AE source is computed at the artificial boundaries of the FE zone. Decompositions at left and right of the source, given in Fig. 8 as functions of frequency over the receiver bandwidth ([0.16 – 0.23] MHz) differ: emitted waves interact with crack and plate surfaces differently before they propagate in both parts of the plate.

SUMMARY AND PERSPECTIVES

A modeling approach for GW / NDT simulation has been described; wavefields in

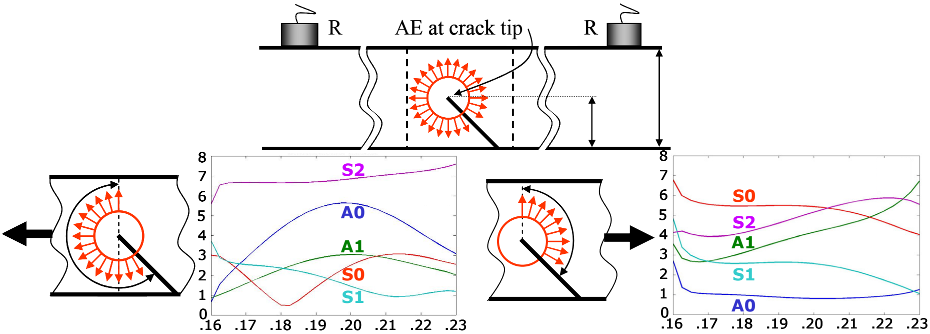

FIGURE 9. Top: AE configuration considered. Bottom: modal decompositions of the AE source at artificial left and right boundaries of the FE zone as functions of the frequency (in MHz). Mode amplitudes are arbitrary scaled but their relative values are quantitatively comparable.

guides are decomposed on modes and formulas link local and non-local models of phenomena typical of GW measurements. The SAFE method is used for computing long range propagation in uniform guides. GW scattering can be computed in some simple cases (crack normal to the propagation axis) by a scheme derived from SAFE, similarly to transducer diffraction effects. A specific FE method has been derived for computing the scattering by arbitrary inhomogeneities; it includes exact transparent artificial boundaries for reducing the size of the FE zone, thus the computation costs. Overall modal formulas offer many post-processing capabilities that help data interpretation as illustrated by some examples. Some of these tools will be implemented in future versions of CIVA software platform [11]. Experimental validation in complex cases, applications to non-destructive testing methods such as SHM or AE are in progress.

REFERENCES

1. P. Cawley, M. J. S. Lowe, D. N. Alleyne, B. Pavlakovic and P. D. Wilcox, *Mat. Eval.* **61**, pp. 66-74 (2003).
2. J. L. Rose, *J. Pressure Vessel Technol.* **124**, pp. 273-282 (2002).
3. K. Jezzine and A. Lhémery, “Simulation of Guided Wave Inspection Based on the Reciprocity Principle and the Semi-Analytical Finite Element Method”, in *Review of Progress in QNDE,* **26A**, edited by D. O. Thompson and D. E. Chimenti, AIP Conf. Proc. vol. 894, American Institute of Physics, Melville, NY (2007), pp. 39-46.
4. B. A. Auld, *Wave Motion* **1**, pp. 3-10 (1979).
5. T. Hayashi, W. J. Song and J. L. Rose, *Ultrasonics* **41**, pp. 175-183 (2003).
6. K. Jezzine and A. Lhémery, “Diffraction Effects on Ultrasonic Guided Waves Radiated or Received by Transducers Mounted on the Section of the Guide”, in *Review of Progress in QNDE*, **25A**, *op. cit.* (2006), pp. 134-141.
7. A. Lhémery and K. Jezzine, “Two Efficient Methods of Mode Selection in a Waveguide Insonified by a Transducer Mounted on its Section”, in *Review of Progress in QNDE*, **27A**, *op. cit.* (2008), pp. 123-130.
8. J. Li and J. L. Rose, *J. Acoust. Soc. Am.* **109**, pp. 457-464 (2001).
9. V. Baronian, A.-S. Bonnet-BenDhia and É. Lunéville, accepted for publication in *J. Comput. Appl. Math.* (2009).
10. V. Baronian, A.-S. Bonnet-BenDhia, A. Lhémery and É. Lunéville, *Springer Proc. Phys.* **193**, pp. 137-147 (2009).
11. Details on present capabilities of CIVA software platform available in www-civa.cea.fr